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Biomechanics of Diarthrodial Joints: A Review of Twenty Years of Progress

A survey of some of the advances made over the past twenty years in understanding diarthrodial joint biomechanics is presented. Topics covered in this review include biotribology (i.e., friction, lubrication and wear of diarthrodial joints);contact area determinations; stereophotogrammetric rendering of articular surfaces; deforе motional field analysis using canonical problems; and finite element formulations for both infinitesimal and finite deformations of biphasic materials and precise anatomic surfaces. Suggestions are made for future research directions as well.

Introduction

Diarthrodial joints are the intricate mechanisms that provide between skeletal segments of animals. These motions allow locomotion (lower extremity function) and activities of daily living (e.g., hand function) to occur. Historically, biomechanical studies of joints were based on the need to better understand their structure-function relationships in providing joint motion, and the patho-mechanical processes involved in joint diseases such as osteoarthritis [l, 2]. These studies have evolved significantly in the last twenty years as advances in theoretical analyses, experimental methodologies and highspeed computing methods have provided new opportunities for progress [3].

The object of this review is to present some of the important gains which have taken place in the past two decades in biotribology, quantitative descriptions of joint anatomy, joint contact and cartilage stresses, and finite element modelling of diarthrodial joints. Other areas relating to diarthrodial joints, e.g., tendons, ligaments, bone, etc., are also covered in this special JBE volume.

Biotribology

Tribological studies have been applied to every conceivable bearing system made by man. Various theories have ben proposed for the study of lubrication of bearings. The most fundamental ones are the hydrodynamic lubrication theory of Reynolds (1886) elasto-hydrodynamic lubrication theory of Dowson and Higginson (1977) and boundary lubrication theories (see Bowden and Tabor, 1964 [61; Peterson and Winer (1980) [7]). These modes of lubrication also operate within diarthrodial joints.

Under healthy conditions, diarthrodial joints function in a nearly frictionless and wear resistant manner throughout life. Failure of these bearing surfaces (i.e., articular cartilage), as with engineering bearings, means a failure of these bearings to provide their essential lubrication function. In biomedical terms, failure of diarthrodial joints leads to osteoarthrosis, or simply OA. While the depiction of the arthritic process in joints as the failure of the lubrication process in bearings is an over—simplification, the analogy between an engineering bearing and a diarthrodial joint is apt. In these joints, synovial fluid, articular cartilage and supporting bone are the essential materials forming the bearing system. The performance of these bearings depends on the mechanical behaviors of the materials comprising the joint. The reader is referred to the review in this volume by Lai et al., on progress made over the past 20 years on constitutive modeling of these materials. In this paper, only the performance of these bearings, i.e.,the friction, lubrication and wear characteristics, and methods required to model these bearings will be reviewed. From the engineering view, benefits could also be gained from an appreciation of nature's bearing design.

All diarthrodial joints have some common structural features, Figs. I(a) and (b). First, they are enclosed in a strong fibrous capsule which is often reinforced by ligaments, e.g., the knee ligaments. Second, this capsule is lined with a metabolically active tissue, i.e., the synovium, which secretes the synovial fluid, long thought to be the lubricant for diarthrodial joints [4]. Third, the articulating bone ends in the joint are lined with a thin layer of cartilage which is a hydrated-soft tissue. These linings, i.e., the synovium and the two cartilage layers, form the joint cavity which contains the synovial fluid, Fig. I(b). The synovial fluid also provides the nutrients required by the chondrocytes residing in the avascular cartilage. Thus the synovial fluid, articular cartilage, and the supporting bone form a closed bearing system which provides the near frictionless bearing system of the body.

The human hip or knee joint may sustain loads up to 10 times body weight during daily activities such as walking [8]. In the hip these forces may yield compressive stresses as high as 18 MPa between a metallic endoprosthesis and the acetabulum during such mild activities as rising from a chair [9]. Even during standing, loads several times body weight are commonly found in the knee and hip. These high loads, and

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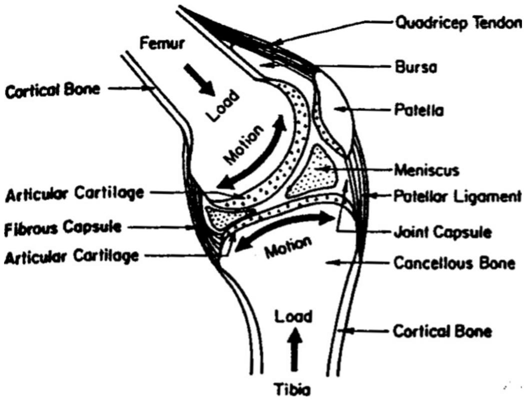
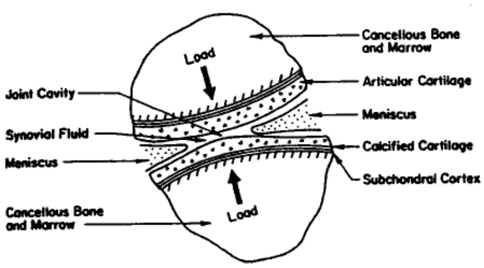


Human Knee Joint

Fig. 1 (a) Diagram of sagittal cross section of a human knee showing structures within the joint including articular cartilage, Joint cape meniscus and bone



Load Bearing Region

Fig.1(b) Close-up view of the joint cavity within the load-bearing region showing the joint cavity and synovial fluid along with the articular cartilage and subchondral bone

therefore high contact stresses, stem from the mechanical disadvantage of the human joints, and can occur even in upper extremity joints, e.g., shoulder. These performance characteristics demand efficient lubrication processes to minimize friction and wear of cartilage in the joint.

Many techniques have been devised to measure the frictional coefficients of joints and wear properties of cartilage [10-18]. A variety of fluid-film lubrication theories, i.e., hydrodynamic theory [4], elastohydrodynamic theory [5, 19] and biphasic squeeze-film lubrication [20], have been used to describe the lubrication process and the extraordinarily low wear properties of diarthrodial joints [10, 13, 15]. Boundary lubrication [6, 7] between the two rubbing surfaces [12], each with an adsorbed monolayer of a lubrication glycoprotein (a component of synovial fluid) [21], and adsorbed layer of "structured-water" [17] have also been proposed to explain the low coefficient of friction. At present, however, no comprehensive or consistent theory exists for diarthrodial joint lubrication under all operating conditions. Considering the complex material behaviors of cartilage and synovial fluid, and the complex loading conditions, it is unlikely that one single lubrication mode exists in diarthrodial joints. However, with increased knowledge of the many components of diarthrodial joints, i.e., articular cartilage [1], and synovial fluid [22], and advances made in quantitative anatomy of the articulating surfaces [23, 24] and the kinematics and load bearing characteristics of these joints [8, 9], the field is now poised to make major advances in understanding joint lubrication, load support within cartilage and the etiology associated with failures of animal bearings. Further, these advances are likely to be accomplished specifically on a joint by

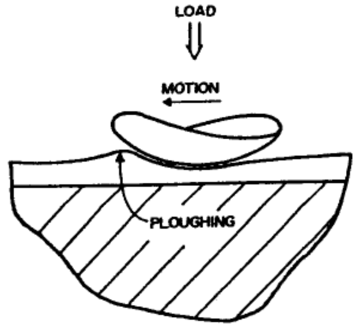


Fig. 2 Illustration of ploughing effect between a hard smooth surface (top) and a soft layered medium supported by a rigid foundation [1]

joint basis, i.e., hip, knee, shoulder, wrist, etc. [24-26].

Coefficient of Friction. Jones (1936) was one of the first to report on the coefficient of friction in animal joints (1 l). Since then many studies on friction and wear have been performed and reported in the literature [12-17]. For all joints tested, the coefficient of friction was very low, ranging from 0.002 to 0.35. The latter number was an exception to all reported data and was obtained when the cartilage fluid content was allowed to be greatly diminished [13]. In general, fully hydrated cartilage loaded under dynamic (oscillatory) conditions, and articular surfaces lubricated with synovial fluid, had a coefficient of friction less than 0.01. Clearly, animal joints enjoy a very low coefficient of friction when compared to common engineering bearings. Paradoxically, diarthrodial joints seem to demonstrate a higher kinetic coefficient of friction than static coefficient of friction [14]. This is a significant tribological fact for the function of joints which are always loaded in a cyclical manner. These remarkable frictional characteristics have been a major motivational factor behind the efforts to study cartilage and diarthrodial joint biomechanics over the past twenty years.

Mechanism for Ploughing Friction. Joint cartilage is a soft tissue with a compressive modulus of less than 1.5 MPa, a shear modulus of less than 0.5 MPa and a Poisson's ratio which ranges from 0 to 0.42 [27, 28]. While it is a highly hydrated tissue with a water content ranging from 60 to 85percent, this water occupies molecular-size "pores" with a diameter estimated to range from 30 to 60 Å [13, 29, 30]. Most of these pores are "open" and "connected" allowing fluid seepage. When cartilage is deformed the water in the tissue will flow. This fluid flow is resisted by a high drag force because the normal permeability is extremely low, i.e., $0(10^{-15} Sm^4/Ns)$. When a joint moves, ploughing of cartilage occurs (Fig. 2). This ploughing deformation causes interstitial fluid flow, and thus internal dissipation and fluid pressurization. This internal dissipation is the mechanism that gives rise to ploughing friction, and the pressurization gives rise to load support.

Lubrication Theories for Animal Joints. Many frictional experiments have been performed attempting to determine the role of synovial fluid or its components in joint lubrication [12-17]. This has been difficult to do since the deformation and flow properties of cartilage and synovial fluid differ sig-nificantly with loading conditions, and the anatomic form of joints. Further, the study of flow of the non-Newtonian synovial fluid [22] in a thin-film gap between two porous-permeable hydrated soft tissues has many formidable theoretical difficulties[1]. These unresolved difficulties have led to speculations, and incomplete and sometime wrong theories on how fluid-film lubrication may be formed in diarthrodial joints, e.g., hydrodynamic [4, 10], weeping—interstitial fluid flow

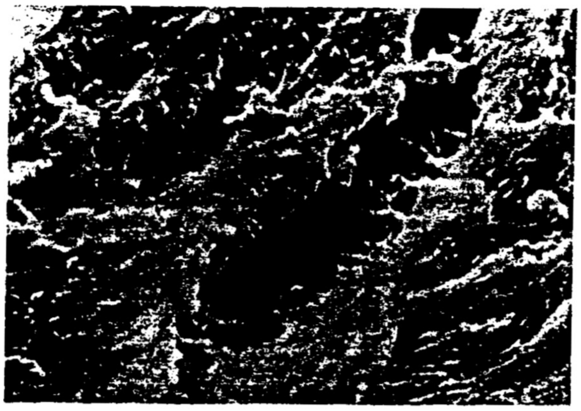


Fig. 3(a) A scanning electron micrograph at X3000 of a osteoarthritic human cartilage removed at surgery showing a micro-crack, (From Mow, V. C., and Soslowsky, L J., Basic Orthopaedic Biomechanics, V. C. Mow, and W. C. Hayes, eds., Raven Press, New York, 1991, Chapter 6.)

Fig.3(b) A scanning electron micrograph at X1000 for a osteoarthritic human hip joint cartilage showing delaminatlon of the superficial tangential zone of the tissue (From Mow, V. C., and Soslowsky, L. J., Basic Orthopaedic Biomechanics, V. C. Mow, and W. C. Hayes, eds., Raven Press, New York, 1991, Chapter 6.)

out of cartilage [13], boosted—filtration flow into cartilage[15], squeeze-film—no flow in cartilage [16] , elasto- and microelastohydrodynamic—no flow in cartilage [19]. One problem has been solved by two groups utilizing an identical squeeze-film formulation of a thin viscous fluid layer between an impermeable rigid spherical indenter and a biphasic layer of cartilage [31, 32], though somewhat surprisingly different conclusions were reached. To date, no problems addressing fluid-film lubrication in a configuration modelling diarthrodial joints with two layers of articular cartilage with different biphasic properties, non-Newtonian synovial fluid properties, precise anatomic forms, and physiologic loading conditions have been solved. Even after the significant progress made over the past 20 years, these problems remain major challenges in biomechanics and biomedical research.

Boundary lubrication experiments were undertaken to examine the molecular aspects of synovial fluid lubrication [21]. In these studies, synovial fluid was treated with various degradative enzymes to digest the proteins and hyaluronate chains and/or passed through fine-pore filters with various pore sizes to eliminate these macromolecules from the fluid. The components from the separation procedure were subsequently tested for their ability to reduce friction. From these studies, a glycoprotein was found which acts as an effective boundary lubricant. Also, it was found that the lubrication properties of synovial fluid treated with hyaluronidase differed from untreated synovial fluid. These two results suggest that a mixed lubrication process might occur within joints where the non-Newtonian fluid plays a role in fluid-film lubrication and the lubricating glycoprotein plays a role in boundary lubrication. Since our joints operate at high loads and slow speeds, it is unlikely that a sufficiently thick fluid film can be generated by any of the classical fluid-film lubrication mechanisms (e.g., hydrodynamic or elasto-hydrodynamic modes) to permit appreciable load carrying capacity. The literature does argue that the extremely low coefficient of friction prohibits ruling out fluid-film lubrication. How fluid-film lubrication can be achieved in a diarthrodial joint is still an issue that needs to be resolved.

Wear of Articular Cartilage. It is estimated that a human knee or hip joint may experience one million cycles of loading per year. These high cyclical stresses and strains may cause fatigue micro-cracks on the articular surface or within the bulk material, and these may grow and accumulate into microscopically observable damages to the articular surface. Figure 3(a) is a scanning electron micrograph (SEM) showing a micro-crack on the surface of human cartilage removed during surgery. Collagen fibers spanning the crack tips are clearly visible, It is likely that the collagen fibers at the articular surface serve as crack arresters. Micro-cracks can be formed within cartilage These micro-cracks can coalesce, eventually causing delamination of the surface membrane from cartilage. Figure 3(b) is a SEM showing a surface layer being removed from the surface of a human hip joint cartilage. In time, if the rate of damage exceeds the rate at which the cells can repair the tissue, an accumulation of such damages could lead to tissue failure [1, 2]. Thus the fundamental difference between wear of a biological material such as cartilage in vivo and wear of an engineering bearing is that in the biological system, there is a balance of mechanical attrition and biological repair.

Wear from cartilage rubbing against cartilage is not the result of adhesive wear since cartilage surfaces do not weld together as do metal surfaces when their micro-asperities come into contact [6, 7, 18]. In some clinical situations, cartilage must articulate against a metal endoprosthesis when one side of a damaged joint is replaced. A study of wear of dead cartilage rubbing against a smooth metal surface has been reported [181. Loss of collagen from the tissue was used as the indicator of wear rate. It was found that even at moderately high pressures, the wear rates were generally very low (<0.5 μg/hr at 4.62 MPa). This low wear rate may be due to the tenacious lubricating glycoprotein serving as the boundary lubricant. However, the mechanisms of this wear process which cause tissue loss are obscure (e.g., delamination?). Finally, as with engineering bearings, chemical events (e.g., corrosion) may be very important. In joints, enzymes such as collagenase and metalloproteinase may cause degradation of cartilage. These enzymatic activities are critical once inflammation sets in and, as with corrosive effects on metallic bearings, they act to accelerate the wear and tear processes in cartilage.

Joint Mechanics: Contact Areas and Cartilage Stresses

While the discussions above have focused on bio-tribology, other aspects of joint biomechanics are equally important. Clearly, loading of joints leads to high contact stresses in the supporting joint structures such as cartilage, bone, tendons, and ligaments. High stresses in these tissues, or alterations in the stress patterns resulting from injury or repetitive loading, are believed to be responsible for cumulative tissue damage and OA [2]. Consequently, another major endeavor of the last two decades has focused on determining contact areas and stresses in joints, using experimental methods. Most of these studies dealt with lower-extremity joints, particularly the knee, hip and ankle, although interest in upper-extremity joints (wrist, elbow, and shoulder) is evident in the recent literature. A number of techniques are summarized below, with a particular emphasis on studies of the knee and hip.

Radiographic and Sectioning Techniques. The earliest studies of diarthrodial joint contact were performed radiographically, where "points" of contact were determined. In his 1955 book, Steindler [33] reported on the early radiographic studies dating back to Zuppinger (1904)[34] on the tibiofemoral Radiographic techniques have also been employed in conjunction with radio-opaque solutions injected into the joint, Kettelkamp and Jacobs, 1972 [35]; Maquet et al., 1975 [36]). Sectioning techniques have been used to determine joint contact areas. Wiberg (1941) sectioned frozen knee joints to analyze contact in the patellofemoral joint [37]. While this technique can be used for only one joint position, the results reported by Wiberg were in agreement with subsequent studies using other methods.

Dye Staining Techniques. Greenwald and O'Connor (1971) determined contact areas in the hip joint using a dye staining method (38). In this method, the cartilage surfaces are first allowed to absorb a chemical agent prior to applying a load on the joint. While the joint is loaded, it is exposed to a second liquid chemical agent which reacts with the first agent in cartilage to produce a colored stain in all those regions of the articular surfaces not in contact. Subsequently, the joint is washed with normal saline and the surface is removed to allow viewing of the staining. A similar approach was employed by Goodfellow et al. (1976) [39] for studies of the knee joint and Moran et al. (1985) for studies of the finger joints [40]. Matthews et al. (1977) applied methylene blue on the retropatellar surface prior to loading the joint, and then assessed the size and location of the imprint of the patella on the opposing trochlear surface [41].

Casting Techniques. In this method, a casting material is injected into the joint while still a liquid, either prior or subsequent to load application. In the former case, the casting material is squeezed out of the regions of contact, thereby creating holes or translucent regions in the hardened cast. In the latter case, the cast will only contain holes, which correspond to regions where the material could not penetrate. The most common casting materials used are methylmethacrylate and silicone rubber. Probably the earliest casting study was performed by Walker and Hajeck (1972) on the tibiofemoral joint [42]. Recently, Yao and Seedhom (1991) introduced the "3S technique," where a silicone oil-carbon black powder suspension is squeezed out from the regions of contact between the joint surfaces following loading [43].

Surface Proximity Techniques. Scherrer et al. (1979) introduced a joint contact determination method based on the calculation of the relative proximity of the articular surfaces at various joint positions [44]. In this method, mathematical models of the articular surfaces are created from 3-D surface measurements [45], and the relative positions of these surfaces at various joint angles are measured with a kinematic linkage. Using the kinematic data, the mathematical surfaces are realigned to assume their relative in situ positions, and the distances between the surfaces calculated. All those regions of the opposing surfaces which fall within a prescribed distance from each other are defined as contact areas. A similar analysis was performed by Soslowsky et al. (1992) [26] using close-range stereophotogrammetry (SPG) for surface topography [23-25], and kinematic measurements. Most recently, Kwak a et al. (1993) reported on the contact areas and cartilage thickness distribution in greyhound patellae, distal femurs and tibial plateaus using SPG [46]. Figure 4(a) demonstrates the relative position of the patella and distal femoral articular surfaces at femoral articular surfaces at

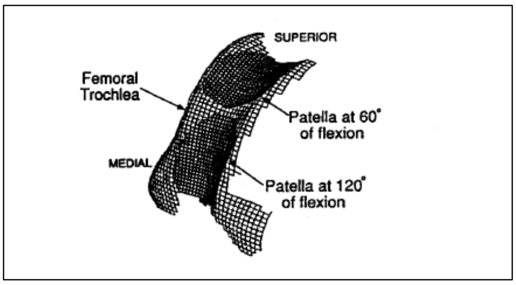


Fig.4(a) Typical configuration of the relative position of the greyhound patella and distal femoral articular surfaces at 60 and 120 degrees of flexion, as obtained with stereophotogrammetry

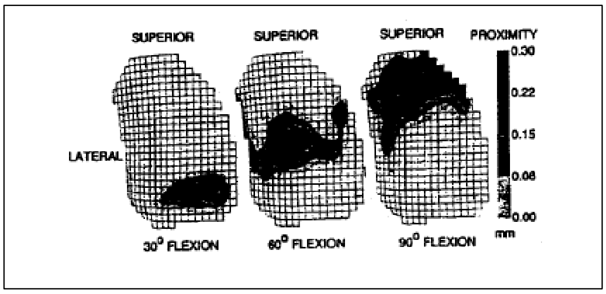


Fig. 4(b) Patella contact areas as proximity maps, at 30, 60, and 90 degrees of flexion

60 and 120 degrees of flexion, while Fig. 4(b) displays the contact areas on the patella at 30, 60, and 90 degrees.

Pressure Measurement Techniques. In addition to measuring contact areas, investigators have also been interested in measuring contact stresses or pressures. Ferguson et al. (1979) measured contact stresses in the patellofemoral joint using miniature piezoresistive contact pressure transducers implanted at various locations on the retropatellar cartilage [47]. This type of transducer can be used for measuring transient stress responses at discrete locations in the joint, providing a history of the time-dependent response of cartilage which is known to behave in a viscoelastic manner. Piezoresistive transducers were also used by Brown and Shaw (1983) to look at contact stresses in the hip [481 and on the femoral condyles [49].

Fukubayasi and Kurosawa [50] used pressure sensitive film (by Fuji Film Co., Ltd., Tokyo) to measure contact pressures in the joint directly, by inserting the film between the contacting articular surfaces. This pressure sensitive film consists of two polyester sheets which, when pressed against each other, will produce a red stain whose intensity depends on the applied pressure. As the film is sensitive to humidity, it is generally sandwiched between polyethylene sheets; under such a configuration, the film is approximately 0.2 to 0.3 millimeters thick. To assess the actual pressure from the intensity of the red stain, a calibration procedure is necessary [e.g., 51]. Because of its simplicity, the Fuji pressure-sensitive film has been used extensively; unfortunately not all of it could be referenced here. To mention a few, however, in the area of knee research for example, Huberti and Hayes (1984) used pressure sensitive film to analyze the influence of Q-angle and tendofemoral contact [52] as well as the effects of capsular reconstructive procedures [53] on patellofemoral contact pressures. Similarly, Brown et al., studied contact stress aberrations in the presence of osteochondral defects (1991) [54]; Haut measured patellofemoral contact pressures during impact loading of the knee [55]. A similar technique was employed by Ahmed (1983) who studied pressure distributions on the tibia [56]and the retropatellar surface [57]. Recently, a comparison study of four of the above techniques has been completed (dye staining, silicone rubber casting, Fuji pressure-sensitive film, and SPG) (1993) [58]. It was found that dye staining could over-estimate contact areas while silicone rubber casting tended to underestimate contact areas, particularly in highly congruent articulations; Fuji film and SPG provided very consistent results.

In vivo measurements of contact areas or pressures in normal human joints have not yet been reported, although one study has employed an instrumented metallic hip endoprosthesis to measure the contact pressures [9]. To date, there appears to be only one study which has reported on measuring compressive strains in hip joint cartilage in situ [59]. It was found that for normal hip joints, compressive strains do not exceed 15 percent.

Theoretical Analyses of Joint Mechanics. In contrast to experimental studies, few theoretical models of joint contact exist in the literature. Hirsch (1944) proposed to use the Hertz contact theory for contacting elastic spheres to model cartilage indentation [60]. Askew and Mow (1978) analyzed the problem of a stationary parabolically-distributed normal surface traction acting on a layered transversely isotropic elastic medium to assess the function of the stiff surface layer of cartilage [61]. More recently, Eberhardt et al. (1990, 1991) [62, 63] developed a solution for the contact problem of normal and tangential loading of elastic spheres, with either one or two isotropic elastic layers to model cartilage. Using the biphasic model for cartilage [64], Mow and Lai (1980) calculated stresses in a cartilage layer subjected to a moving, parabolically-distributed normal surface traction [65]. The load partition factor between the solid and fluid at the articular surface was used in their parametric analysis. Armstrong (1986) studied the contact problem between a cylinder and an elastic layer resting on a rigid foundation, using a thin layer asymptotic analysis for a compressible (equilibrium response) and incompressible (initial response) material [66]. Most recently, Ateshian et al. (1992, 1993) performed an asymptotic analysis of the contact of two spherical isotropic biphasic layers subjected to a step normal load subsequent to the depletion of the thin lubricant film between the surfaces [67, 68]. The results from this analysis were in agreement with the solution of a squeeze-film problem addressing the initial approach of the surfaces [20]. The results from the studies listed above have demonstrated that considerable differences may be found in cartilage stress predictions depending on the particular cartilage constitutive model being employed, e.g., linear elasticity theory versus linear biphasic theory. Other studies have made use of joint mathematical models which incorporate realistic geometric data, to predict contact areas and stresses in joints; the reader is referred to the studies by Wismans et al. (1980) [70], Blankevoort et al. (1991) [71], and Hirokawa (1991) [72].

Finite Element Modeling of Joints

Joint biomechanics problems are characterized by moving contacts between two topographically complex soft tissue layers separated by a thin layer of non-Newtonian synovial fluid. Examples of this type of problem include the sliding contact between the humeral head and the glenoid in the shoulder or the patella over the femoral groove in the knee, and the multibody contact problem between the tibia, femur and menisci. The complexity of such problems requires implementation of sophisticated numerical methods for solutions. The finite element method is ideally suited for obtaining solutions to joint contact problems. Thus far, much of the finite element analysis has been applied to the study of hard tissue structures, often as it relates to prosthetic devices. The focus of our discussion of computational mechanics in this review is on diarthrodial joint analysis.

Joint Contact Analysis. As a consequence of the relative dearth of precise geometric data and material properties, as well as the complexity of a properly formulated joint mechanics problem, few such computational models have been reported. Two-dimensional studies of the tibial-femoral contact problem in the knee [73], and in the hip [74, 75] have been repotted Heegaard (1993) presented a 3-D contact analysis of the knee assuming perfectly lubricated interaction between tissue layer and allowing for large relative motions of the layers [76]. In each of these studies, the soft tissue layers are treated as single phase elastic materials.

Analysis of the Meniscus. The meniscus of the human knee has been studied via finite elements. Initially, axisymmetric toroidal models of the meniscus were developed assuming frictionless contact between the meniscus and deformable femur and tibia and single phase elastic materials [77, 78] or transversely isotropic materials [79]. Aspden (1985) used nonlinear tensile properties in the circumferential direction and linear compressive properties in the cross section (chosen to match the cartilage layer properties) in an axisymmetric model resting on a rigid frictionless tibial surface with loads applied to the femoral surface [80]. Three-dimensional models of the meniscus including contact and single phase material representations allowing for different properties in the circumferential direction have also been reported [81]. An axisymmetric transversely isotropic biphasic model of the meniscus, supported by a frictionless tibial surface and loaded through the femoral surface, has been developed by Spilker et al. [82]. Important insights into meniscal behavior have been obtained from these models. However, significant effort remains to incorporate the meniscal analyses into a more complete mechanics representation of the three-body contact problem of the knee.

Future Directions in Finite Element Joint Analysis. To tackle the problem of joint analysis, fundamental studies will be needed in the following areas: l) 3-D finite element analysis of moving contact problems utilizing nonlinear finite deformation biphasic laws for cartilage and non-Newtonian laws for synovial fluid; 2) automated-adaptive methods for the generation and control of 3-D computational models using error estimates and controls which account of the nonlinearities, singularities and boundary layer effects; 3) integration of digital SPG anatomic data with the automated adaptive 3-D mesh generation, and numerical methods for processing the material and geometric data required for the contact analysis; and 4) parallel solution algorithms for the nonlinear time-dependent problems utilizing high-performance computer architectures.

It is widely accepted that the time-dependent response of cartilage can be accurately represented by the biphasic theory derived by Mow et al. (1980) [64]. Under isotonic conditions, this biphasic theory of incompressible solid and fluid phases is appropriate for most applications involving cartilage modeling for infinitesimal or finite deformations. The mathematical problems associated with these studies are therefore posed in terms of partial differential equations (continuity equation for the mixture, momentum equations and diffusive momentum exchange for each phase, and nonlinear constitutive relations), initial and boundary conditions, and interface conditions. These interface conditions now exist for a biphasic surface in contact with a fluid [69], or against an impermeable or permeable solid [20], or against another biphasic surface [67, 68]. Numerical methods are required to solve these nonlinear problems, even for relatively simple geometries. To be useful, these solutions will ultimately be needed in real time.

Finite element formulations corresponding to the nonlinear biphasic mixture theory have been developed using alternate approaches which include 2-D applications to both soft tissues and soils, for example, (Prevost (1982), [83]; Suh et al. (1991), [84]; Wayne et al. (1991), [851; Spilker and Almeida (1992), [86]). Simon and Gaballa (1988), [87] have developed nonlinear finite element formulations for hydrated tissues based on the poroelastic consolidation theory of Biot (1941), [88]. For cartilage, where inertial effects have been neglected, these finite element formulations yield a system of coupled first order differential equations with nonlinear coefficient matrices, whose solution is obtained through incremental and iterative methods. Solution of these equations provides a detailed description of fluid flow, strain, stress and pressure fields within the layer of tissue. Linear 3-D elements [89] and nonlinear 3-D formulations [90] have been presented recently for the biphasic theory. However, full nonlinear 3-D analysis for joints of realistic geometry remains a computationally challenging problem.

The moving contact problem between two layers of biphasic tissues will require that contact conditions be incorporated into a finite element formulation. This problem is intrinsically nonlinear since the extent of the contact surface at a given level of load or prescribed joint displacement is not known a priori. For elasticity problems, the concept of a contact element, whose degrees of freedom correspond to contact traction, has been successful and has been used recently by Heegaard (1993), [76] in a 3-D elastic contact analysis of the patellofemoral articulation. An analogous approach is being developed by Donzelli and Spilker (1993) for biphasic materials [91].

In order to apply these computational methods to joint mechanics problems, precisely measured anatomic data, e.g., from the SPG method, should be used to construct a 3-D solid model from which an appropriate finite mesh can be constructed and analysis performed. A procedure for using SPG data as the anatomic input for a 3-D finite element biphasic model is under development in our laboratories (Spilker et al. (1993), [92]); a representative example currently under study will help to illustrate the process and level of complexity. The SPG data for a human humeral head consist of the coordinates defining the collection of bilinear patches making up the upper and lower surfaces of the tissue [26]. The Finite Octree mesh generator [93], using newly developed operators to interface with the SPG data, is used to subdivide the domain into 10-node tetrahedral elements, and attributes (boundary conditions) are assigned to the model faces (or portions of those faces). This 3-D biphasic finite element analysis has been performed using a 3-D biphasic mixed-penalty element [89]. For the humeral head problem a relatively coarse mesh (1303 elements, 2463 nodes, 6 unknowns per node) has been constructed with increased refinement in the vicinity of a centrally located, circular shaped, parabolically distributed load on the upper surface. This preliminary study helped to illustrate the magnitude of effort and the essential elements required to develop a 3-D finite element analysis of a diarthrodial joint using the biphasic theory.

In a contact problem, refined meshes are needed in the vicinity of the contact zone. For biphasic materials, mesh refinement is often required in the vicinity of loading. Thus the mesh generator must have the *intelligence* to place elements optimally, i.e., using more elements where needed and fewer elements away from critical locations. This process may also be time dependent as the contact zone and/or load change with time. Mesh adaptivity relies on a measure of the error which can be used to identify regions where refinements or coarsenings are needed. Rigorous error measures or reliable error indicators must be defined and proven for the finite element formulation of the nonlinear biphasic equations. Using an error measure based on the distribution of total stress in a biphasic material, Donzelli et al. (1992) have examined several linear 2-D axisymmetric biphasic problems [94]. Figure 5 shows four meshes generated for an axisymmetric representation of the meniscus resting on a lubricated tibial surface and loaded

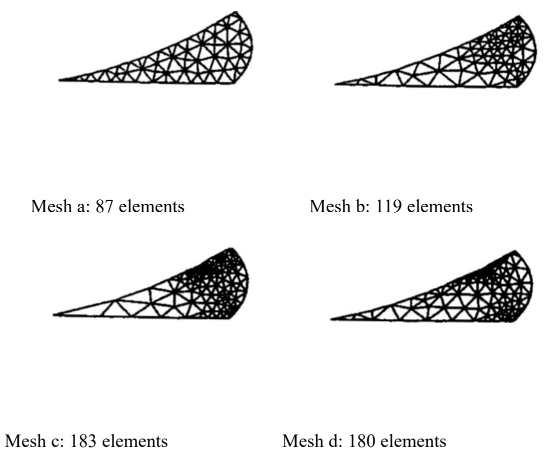


Fig. 5 Four meshes generated for an axisymmetric representation of the meniscus. The mesh refines in a region of high stress gradients and coarsens where stress gradients are less severe [82]

on the full femoral surface by a parabolically-distributed load, and using a transversely isotropic solid phase in the biphasic model. The mesh is automatically refined in a region of high stress gradients and coarsened where stress gradients are less severe [82].

Conclusion

This brief review of research in diarthrodial joint biomechanics is a testimony to the exciting progress that has taken place in this field over the last few decades, while at the same time it provides a sobering perspective of the complexities of the various mechanisms responsible for the proper functioning of diarthrodial joints. Each of the three areas selected in this review, namely biotribology, joint contact mechanics and finite element modelling of diarthrodial joints present major challenges that need to be addressed in future research.

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